

Parallel Lines :

May Nakoua (Only definition)

* Parallel lines in 2D are two lines that never meet regardless the direction!

* Parallel lines in 3D are two lines that never intersect + have the same direction!

* In 3D, if two lines don't intersect but have different directions they're called non-intersecting lines

* The terms of L_1 & L_2 being parallel in 3D

(1) D_1 (directed vector of L_1) & D_2 (directed vector of L_2) have to be a multiple of the other.

$$D_2 = C D_1$$

↓
constant

* if they're equal, they're the same line, so the intersection is the whole line.

(2) They do not intersect!

Question: if $L_1: \begin{cases} x = 2 + 3t \\ y = -2t \\ z = 1 + 3t \end{cases} \quad \left. \vphantom{\begin{cases} x = 2 + 3t \\ y = -2t \\ z = 1 + 3t \end{cases}} \right\} \text{ EER}$

$L_2: \begin{cases} x = 12w \\ y = -8w \\ z = 10 + 12w \end{cases} \quad \left. \vphantom{\begin{cases} x = 12w \\ y = -8w \\ z = 10 + 12w \end{cases}} \right\} \text{ WER}$

Is L_1 parallel to L_2 ?

Find D_1 & D_2

$$D_1 = \langle 3, -2, 3 \rangle$$

$$D_2 = \langle 12, -8, 12 \rangle$$

Is $D_2 = c \cdot D_1$

$$\langle 12, -8, 12 \rangle = c \langle 3, -2, 3 \rangle$$

$$12 = c \cdot 3 \Rightarrow c = 4$$

$$-8 = c(-2) \Rightarrow c = 4 \quad D_2 = 4D_1$$

$$12 = c \cdot 3 \Rightarrow c = 4 \quad D_1 = \frac{1}{4}D_2$$

step 1) checking if D_2 is a multiple of the other is Done

step 2) checking if they intersect

choose a point from L_1 by giving t a value

say $t = 0$

$$x = 2 + (3)(0) = 2$$

$$y = -2(0) = 0$$

$$z = 1 + 3(0) = 1$$

$$\therefore \text{point} = (2, 0, 1)$$

Check if this point lies on L_2

$$2 = 12w$$

$$0 = -8w$$

$$1 = 10 + 12w$$

each equation gives a different value of w

$\therefore P(2, 0, 1)$ doesn't lie on L_2

\therefore No intersection between L_1 & L_2

Perpendicular Lines :

In 3D $L_1 \perp L_2$ if only :

(1) $D_1 \cdot D_2 = 0$

(2) They intersect in a point say Q

Question 1) $L_1 : \left. \begin{array}{l} x = 2 + 4t \\ y = -4 - 2t \\ z = 1 + t \end{array} \right\} \text{ tER}$

$$L_2 : \left. \begin{array}{l} x = 10 - 2w \\ y = -3w \\ z = -2 + 2w \end{array} \right\} \text{ wER}$$

Is $L_1 \perp L_2$?

First, Find D_1 & D_2 $D_1 = \langle 4, -2, 1 \rangle$

$$D_2 = \langle -2, -3, 2 \rangle$$

$$D_1 \cdot D_2 = 4 \cdot (-2) + (-2) \cdot (-3) + 1 \cdot 2$$
$$-8 + 6 + 2 = 0$$

$$\therefore D_1 \perp D_2 = 0$$

Notes not complete.
OUR DEFINITION of
PERPENDICULAR, check
for 2 things. $D_1 \cdot D_2 = 0$
and they intersect in a
point.